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Microwave photon detection by an Al Josephson junction

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Abstract

The aluminum Josephson junction (JJ) with a critical current, suppressed by a factor of three comparing with the maximal value calculated from the gap, is experimentally investigated for application as a threshold detector of microwave photons. We present the preliminary results of measurements of the lifetime of the superconducting state and the probability of switching by 9 GHz external signal. We found an anomalously large lifetime, not described by the Kramers' theory for the escape time over a barrier under the influence of fluctuations. We explain it by the phase diffusion regime, which is evident from the temperature dependence of the switching current histograms. Therefore, the phase diffusion allows to significantly improve the noise immunity of a device, radically decreasing the dark count rate, but it will also decrease the single photon sensitivity of the considered threshold detector. Quantization of the switching probability tilt versus signal attenuation for various bias currents through the JJ is observed, which resembles the differentiation between N and $N + 1$ photon absorption.

Keywords

Josephson junction; switching current distribution; phase diffusion; photon counter

Introduction

Currently, an important problem is the creation of single-photon counters in the GHz frequency range. Such devices are in demand in several areas, such as the search for axions, the alleged particles of dark matter [1-3] and quantum computing [4]. Commercially available single-photon detectors operate at frequencies of hundreds of THz and higher [5,6]. For the lower frequency range, a new class of single microwave photon detectors is needed. In this regard, a current-biased Josephson junction is of particular interest for applications as a threshold detector since its phase dynamics is altered even by a weak probe field. Rich dynamics of the JJ constantly inspires new applications, such as thermometry [7,8], noise statistics [9-11] and single photon detection [12].

There are, at least, two different approaches for practical realization of the single-photon detectors based on Josephson junctions, both having their advantages and disadvantages. The first approach relies on a continuous current sweep at a constant repetition rate and the measurements of the switching current distributions, from which the response and sensitivity can be determined [13-15]. In particular, in [15] the tunneling properties of the current-biased Josephson junction coupled with a resonator directly depend on the number of microwave photons in the resonator. The main disadvantages of this approach are the long initialization and freezing times of the detector. The detector works by slowly increasing the bias current from zero. This ramp takes seconds to avoid non-adiabatic excitation in a JJ. As soon as the detector switches, it must be reset by setting the current back to zero and waiting when a Josephson phase relaxes in a potential well. This implies a low repetition rate.

The second approach for experimental microwave detection [16,17] uses the switching events of the biased Josephson junction resulting from a single absorption. In contrast to the previous approach, this one requires less downtime of the detector, determined by the biasing time to the desired current only. However, the operation in this mode does not provide information on the number of absorbed photons and only above-threshold signals can be detected. Also, a special care must be taken to minimize the false switching events of the detector due to thermal fluctuations and macroscopic quantum tunneling.

In this article the second approach is used in application to a prototype of a single photon counter described in [3]. We study the possibility of detecting photons of GHz frequency range using and aluminum Josephson junction with a suppressed critical current. The main requirement to the such counter is an extremely large lifetime (thousand of seconds), orders of magnitude larger than the switching time after the photon absorption (typically less than nanoseconds). In [3] it was shown theoretically that both the required sensitivity and the noise immunity can be reached at the same time in JJ with a suppressed critical current. Besides that, the mesoscopic junctions with low critical currents have received a great deal of interest by themselves, since they exhibit such a phenomenon as the diffusion of the Josephson phase [18-21].

The Josephson phase diffusion in small junctions has been studied both experimentally [22,23] and theoretically [24]. Recently, this regime has been observed also in layered high-temperature superconductors [25]. The significance of this effect depends on the ratio of thermal fluctuations kT, the damping parameter α and the Josephson energy E_J. Here we will consider a small tunnel junction with the thermal noise intensity of $\gamma = k_B T/E_J \geq 2 \cdot 10^{-2}$ and $\alpha > 0.1$, and show experimentally an unusually large lifetime of the superconducting state, which we attribute to the phase diffusion according to [19]. The increase of the lifetime of the superconducting state due to the phase diffusion was also observed in [26] for the similar conditions. On the other hand the phase diffusion is expected to decrease the sensitivity to single photons for the same reason that it improves the noise immunity. To our knowledge so far there are no works dedicated to the role of the of phase diffusion in the response to single photons. In the last section of the article we show the experimental measurements of the switching probability induced by weak microwave signal and discuss some features of the measured response, which may indicate the sensitivity to several photon bunches.

The analysis of the phase-diffusion phenomena is a special case of a general problem of the motion of a Brownian particle in a washboard potential in the framework of the resistively-capacitevely shunted junction (RCSJ) model for the dynamics of the Josephson phase [27,28]. The tilt of the washboard potential is controlled by the bias current I and is defined as $E_J(I/I_C)$, where I_C is the critical current and $E_J = \hbar I_C/2e$. The particle moves along the potential in the presence of friction, whose strength is characterized by $\alpha = \omega_p/\omega_c$, where $\omega_p = (2eI_C/\hbar C)^{1/2}$ is the plasma frequency, $\omega_c = 2eI_C R_N/\hbar$ is the characteristic frequency, R_N is the normal state resistance and C is the capacitance.

The superconducting state of the JJ corresponds to the rest of the particle in one well of the potential. The exit from this metastable state corresponds to the appearance of the finite voltage at the junction. In the case of low damping (but depending also on the barrier height and noise intensity), the particle, jumping over the barrier, gains enough energy to move along the potential in the running state. While if the damping α is sufficiently large, the escape due to the thermal or quantum fluctuations does not necessarily lead to the running state appearance. After an escape event, the particle can move down the potential for several wells and then relax into one of the potential minima [22]. When the barrier and noise are large, the exit from the well and the subsequent retrapping processes may occur many times at a given bias current.

The most evident signature of the phase diffusion phenomenon is the temperature dependence of the switching current distribution [20,29]. For underdamped junctions ($\alpha \ll 1$), the width of the switching current distributions monotonically decreases with decreasing temperature. In the case of moderately damped junctions ($\alpha > 0.2$) the switching dynamics changes due to the phase diffusion: the width of the distribution decreases with increasing temperature. A change in the sign of the derivative of the second moment of the distribution is a reliable indicator of retrapping processes. Another sign of the phase diffusion is an increase in the lifetime of the superconducting state in comparison with the classical Kramers' theory [30,31]. The exit of the particle from the well due to fluctuations does not lead to the instantaneous appearance of a final voltage at the Josephson junction, which can be seen in experiment as an increase of the noise immunity of the system.

The principle of operation of a single-photon counter based on the Josephson junction is the following: at the initial moment of time, the junction is in a superconducting state with bias current I close to the critical one. In standby mode there is no voltage at the junction. An incoming external signal from a photon (current oscillations) can change the state of the system by switching it to a resistive state with a finite resistance value. At the same time the detector may be triggered spontaneously due to thermal fluctuations in the classical region of temperatures and tunneling through the barrier in the quantum case [14,32].

Experimental

Following the line proposed in Ref. [3], the aluminum $Al/AlO_x/Al$ tunnel junction $0.4 \times 2\,\mu m^2$ was fabricated using a self-aligned shadow evaporation technique. Its current-voltage characteristic shown in the inset of Fig. 1 has a well-defined hysteresis. The double voltage gap of the junction is 0.38 mV, corresponding to the critical temperature of Al $T_C = 1.2$ K, the capacitance is $C \approx 0.036$ pF, the critical current density is $3.8 \cdot 10^{-3}$ kA/cm^2 and the normal resistance is $R_N = 2300\,\Omega$, which gives the maximal possible value of the critical current $I_C^{max} = 1.764kT_C/eR_N$ around 80 nA. The measured critical current is $I_C = 28$ nA at the temperature of 20 mK. The damping of the Josephson junction calculated for the measured I_C is $\alpha = 0.24$.

The sample was mounted in an rf-tight box with a superconducting shielding on the coldest plate of Triton 200 dry dilution refrigerator. The dc-bias wires were filtered with feed-through capacitors at the room temperature and RC filters at the 10 mK cryostat plate, minimizing the effect of unwanted low-frequency noise. In order to avoid ground loops the measurement scheme was designed with a single ground.

For the switching current measurements, the bias current of the junction was ramped up at a constant rate of $\dot{I} = 5 \cdot 10^{-8}$ A/s. The voltage was measured using a low noise room-temperature differential amplifier AD745 and was fed to a high-speed NI ADC-card. This signal was used to trigger a fast record of the switching current value. Such a procedure was repeated at least $5 \cdot 10^3$ times at each temperature, and as a result the switching current histograms were compiled in the temperature range between 1 K and 20 mK. For the lifetime measurements, the experimental setup was the same, except that the bias current was set to a predetermined value during the 20 ms to prevent particle excitation caused by a rapid decrease in the barrier, and remained constant until the appearance of a gap voltage due to thermal noise or quantum tunneling. The lifetime measurements were repeated at least 200 times for each value of the bias current.

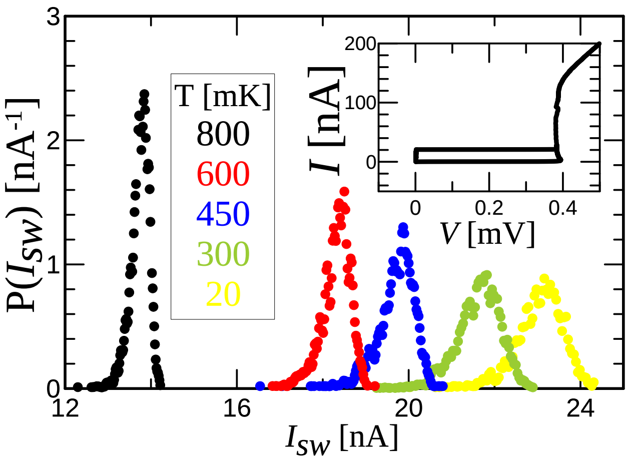

Figure 1: Experimentally measured histogram $P(I_{SW})$ of switching the Josephson junction to the resistive state for current I_{SW} at temperatures indicated from top to bottom for curves from left to right. The inset shows the I-V curve of the junction at 20 mK.

For a high-frequency experiment, a microwave signal was fed into the cryostat via a phosphor bronze twisted-pairs with attenuation of -15 dB per meter at 9 GHz and with a loop antenna near the JJ. The rf-signal from the external microwave synthesizer was attenuated using the voltage controlled room-temperature attenuator, preliminarily calibrated with a commercial spectrum analyzer. The high-frequency signal was varied from a high power at which the Shapiro steps and photon assisted tunneling steps are well pronounced at the IV-curve, to a low power whose presence can be observed only in the switching histograms and in the decrease of the superconducting state lifetime.

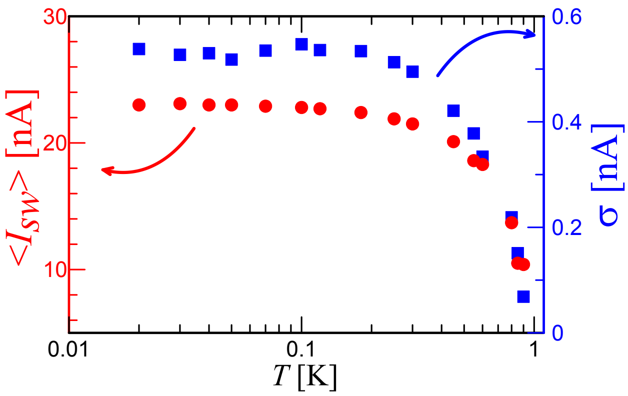

Figure 2: Temperature dependence of the mean switching current (left axis, red dots) and its standard deviation (right axis, blue squares).

Results and Discussion

In this section we present preliminary results of the first measurements. First, we assemble the switching current distributions (Fig. 1) and extract values for the mean switching current $\langle I_{SW} \rangle$ and standard deviation σ, which are plotted in Fig. 2 for various temperatures of the chip. The decrease of $\langle I_{SW} \rangle$ with temperature increase indicates that here the thermal activation of the phase is the main switching mechanism. At the temperatures below $T \approx 300\,\mathrm{mK}$ there is a saturation both in $\langle I_{SW} \rangle$ and σ. The behavior of $\sigma(T)$ in the entire temperature range of the experiment shows the well-known signature of the phase diffusion, observed for example in [20,23,29].

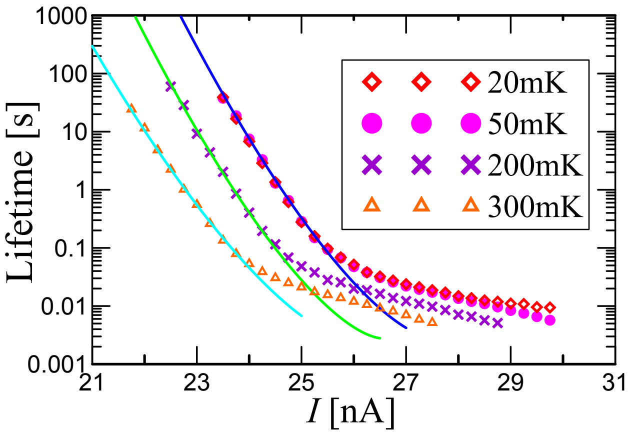

Figure 3: Experimental lifetimes as functions of bias current for various sample temperatures (symbols) and fitting by formula (1) (solid curves).

The presence of the phase diffusion also can explain the results of the lifetime (the inverse of the escape rate) measurements, shown in Fig. 3. The lifetime of the superconducting state corresponds to the mean time of dark counts of a single photon detector. We have measured the dependencies of the lifetime for various bias currents and temperatures and without high-frequency signal. One can see the linear slopes of the lifetime versus bias current for 2-3 orders of magnitude in a logarithmic scale, which means the exponential dependence of the lifetime versus potential barrier height. The plato in the experimental points in Fig. 3 below 0.03 s is due to time constants of the measurement setup. To find out more about the switching conditions the experimental curves have been fitted by the Kramers' formula for the lifetime in the following form [28,30,31] (for the overdamped case, see [33]):

$$\tau = \frac{f(\alpha)\exp(\Delta u/\gamma)}{\sqrt{1-i^2}}, \tag{1}$$

where $i = I/I_C$ is the dimensionless bias current, $\Delta u = 2\sqrt{1-i^2} + i(2\arcsin(i) - \pi)$ is the potential barrier height and $\gamma = I_T/I_C$ is the noise intensity and $I_T = 2ek_BT/\hbar$ is the fluctuational current, which can be calculated for a given temperature T as: $I_T[\mu\mathrm{A}] = 0.042T[\mathrm{K}]$ [27]. As well-known [34], if the well and the barrier of a potential profile can be approximated by parabolas, then $f(\alpha)$ does not depend on the working temperature. However, for the range $\alpha \approx 1$, the exact prefactor $f(\alpha)$ is unknown [31], therefore we use $f(\alpha)$ as a fitting parameter.

Substituting the temperature 300 mK into γ for our experimental parameters, one gets $\gamma = 0.48$. For so large fluctuations the barrier height even with zero bias current is comparable with noise intensity and the corresponding lifetime must be much smaller than measured in the experiment. If we use γ as a fitting parameter together with $f(\alpha)$, we get the best fit for the following parameters: $f(\alpha) = 0.00035$ seconds for all curves, $I_C = 26.5; 27; 28$ nA, noise intensity $\gamma = 0.0137; 0.0112; 0.011$ for temperatures 300, 200, 50 mK, respectively. One can see that I_C in this case corresponds to the measured values.

Thus, the comparison of measurements and fitting shows that the average time between dark counts significantly exceeds the time predicted by Kramers' theory, with mean values reaching hundreds of seconds and thousands of seconds in single measurements. If we believe, that it is the phase diffusion regime significantly suppresses the dark count rate, the next important question is to figure out how it influences the sensitivity to the photons. In order to do this we perform measurements of the detection probability as a function of the attenuator voltage of 9 GHz photons in a 50 ms pulse, incident on the sample area, for three values of bias current I, shown in Fig. 4.

Left vertical axis shows the experimental data i.e., the number of detector counts to the total number of pulses (200 pulses). The horizontal axis corresponds to the attenuation (output power) of the external high-frequency signal. For high incident photon fluxes, the detector switches for all 200 pulses, i.e. counts all pulses. For smaller fluxes our experimental data show that for 2.5 orders

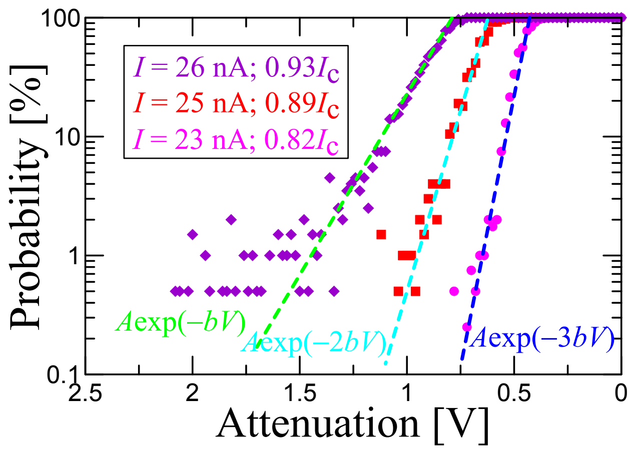

Figure 4: Detection probability of 9 GHz 50 ms pulses of different power (signal attenuation) for different values of the bias current. Dashed lines indicate slopes with exponential factors 1, 2, 3, respectively.

of magnitude, the detection probability decreases linearly (in a log scale) with the decrease of the incident power (average number of incident photons), and the probability slopes for various bias currents are well-fitted by $A\exp(-nbV)$ dependence, and are quantized. Here A and b are fitting parameters and b is the same for all three curves. This resembles the multi-photon detection [5], where for a smaller bias current ($I = 23$ nA), the slope is larger $\approx A\exp(-3bV)$ than for larger bias current $\approx A\exp(-2bV)$ for $I = 25$ nA and $\approx A\exp(-bV)$ for $I = 26$ nA.

Despite we see a consistent switching due to 9 GHz signal even at 23 nA, at the moment we cannot estimate the absorption efficiency, because of the uncertainty in determination of losses in the twisted pair at the frequency 9 GHz and of the absorption efficiency in the junction. Therefore we do not convert the attenuation to the power to avoid an additional insecure parameter. The experiments will be continued with better statistics and signal calibration to extract the number of detected photons. We expect that the sensitivity of the considered threshold detector will be decreased in comparison with the situation without the phase diffusion, however new studies are required to answer this question.

Conclusions

In the present work, temporal and detecting characteristics of a low critical current Al Josephson junction have been studied experimentally. From measurements of switching current distributions and the dark count time intervals, the operation in a phase diffusion regime is evident. It is shown by comparison with the theory that the phase diffusion regime allows to significantly improve noise immunity of a device, radically increasing the mean time between dark counts. However, in the same way, the phase diffusion should decrease the single photon sensitivity of the considered threshold detector, which will be studied in future experiments.

The detection probability versus attenuation voltage shows tail slopes quantization, which resembles a few-photon detection. The use of such a device for supersensitive detection has essential applications. In particular, such a detector can be used in the task of searching the axions and measuring signals generated by quantum circuits at a frequency of 6-9 GHz. In the future, it is supposed to improve the measurement setup and conduct research on the detection of test signals in the range of 8-14 GHz.

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