Supplementary Materials for "Mechanical Property Measurements Enabled by Short Term Fourier Transform of Atomic Force Microscopy Thermal Deflection Analysis"

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1. Equations of Dispersion Relations for Three Cantilever Models

1.1. Solution to Model 1 Dispersion

$$sinh(k_nL)cos(k_nL) - sin(k_nL)cosh(k_nL) = \frac{(k_nL)^3k_c}{3k^*}(1 + cos(k_nL)cosh(k_nL)) \tag{1}$$

1.2. Solution to Model 2 Dispersion

$$\begin{aligned} &-(cosh(k_nL_1)sin(k_nL_1) - sinh(k_nL_1)cos(k_nL_1))(1 + cos(k_nL')cosh(k_nL')) \\ &\quad -(cosh(k_nL')sin(k_nL') - sinh(k_nL')cos(k_nL'))(1 - cos(k_nL_1)cosh(k_nL_1)) \\ &\quad\quad = 2k_n^3\frac{EI}{k^*}[1 + cos(k_n(L_1 + L'))cosh(k_n(L_1 + L'))] \end{aligned} \tag{2}$$

1.3. Solution to Model 3 Dispersion

$$\frac{k^*}{k_c} = \frac{-B \pm \sqrt{B^2 - 4AC}}{6A} \tag{3}$$

$$A = \left(\frac{\kappa}{k^*}\right)\left(\frac{h}{L_1}\right)^2 (1 - cos(x)cosh(xL_1))(1 + cos(xL')cosh(xL')) \tag{4}$$

$$B = B_1 + B_2 + B_3 \tag{5}$$

$$C = 2(xL_1)^4(1 + cos(xL_1)cosh(xL_1)) \tag{6}$$

$$\begin{aligned} B_1 = &\left(\frac{h}{L_1}\right)^2 (xL_1)^3 \left(sin^2(\alpha) + \frac{\kappa}{k_c}cos^2(\alpha)\right) \\ &[(1 + cos(xL')cosh(xL'))(sin(xL_1)cosh(nL_1) + cos(xL_1)sinh(xL_1)) \\ &\quad -(1 - cos(nL_1)cosh(nL_1))(sin(xL')cosh(xL') + cos(xL')sinh(xL'))] \end{aligned} \tag{7}$$

$$B_2 = 2\left(\frac{h}{L_1}\right)(xL_1)^2\left(\frac{\kappa}{k_c}cos(\alpha)sin(\alpha)\right)$$
$$[(1+cos(xL')cosh(xL'))(sin(xL_1)sinh(nL_1))$$
$$+(1-cos(nL_1)cosh(nL_1))(sin(xL')sinh(xL'))] \quad (8)$$

$$B_3 = (xL_1)(cos^2(\alpha)+\frac{\kappa}{k_c}sin^2(\alpha))$$
$$[(1+cos(xL')cosh(xL'))(sin(xL_1)cosh(nL_1)-cos(xL_1)sinh(xL_1))$$
$$-(1-cos(nL_1)cosh(nL_1))(sin(xL')cosh(xL')-cos(xL')sinh(xL'))] \quad (9)$$

$$G^* = \left(\frac{2-\nu_1}{G_1}+\frac{2-\nu_2}{G_2}\right)^{-1} \quad (10)$$

$$G = \frac{1}{2}\left(\frac{E}{1+\nu}\right) \quad (11)$$

$$\kappa = 8G^*a = \frac{8G^*k^*}{2E^*} = \frac{4k^*\left(\frac{2-\nu_1}{\frac{1}{2}\frac{E_1}{1+\nu_1}}\right)+\left(\frac{2-\nu_1}{\frac{1}{2}\frac{E_1}{1+\nu_1}}\right)}{E^*} \quad (12)$$

2. Cantilever Frequency Power Spectrum

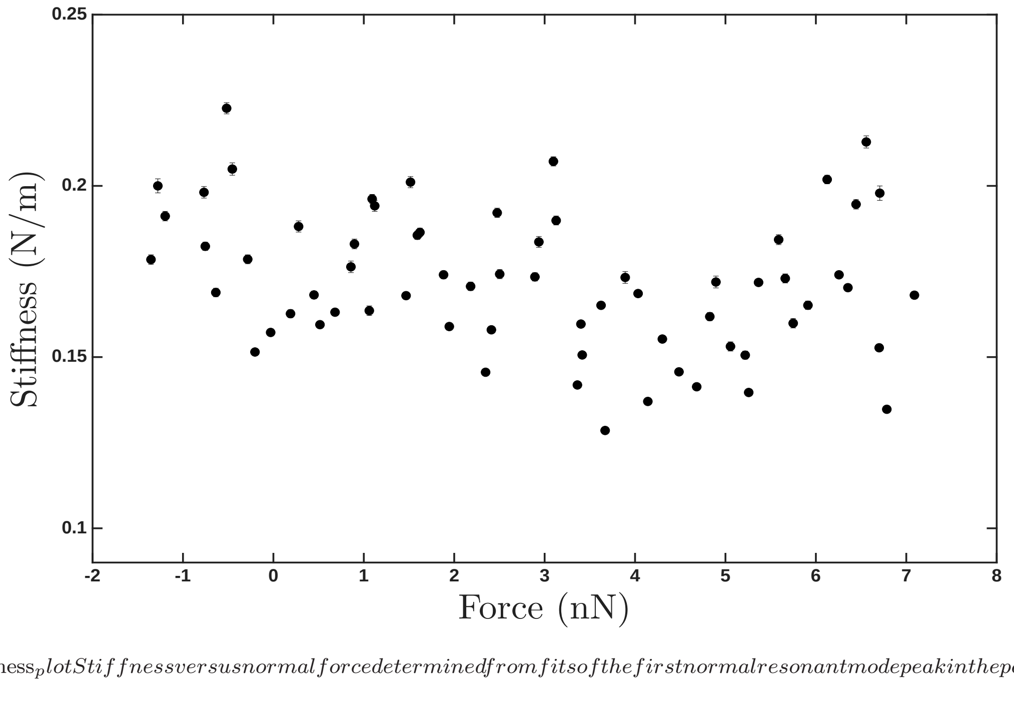

Fig. S1. tiffness$_{plot}$$Stiffnessversusnormalforcedeterminedfromfitsofthefirstnormalresonantmodepeakinthepowerspectraofthecontactpor$

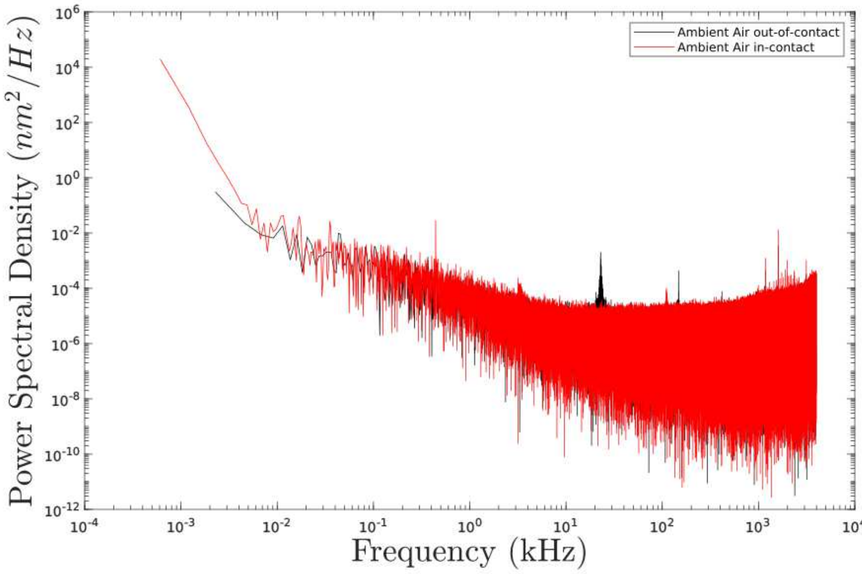

Fig. S2. iamond$_{f}requencyFouriertransformoftheout-of-contactportionofadiamondcoatedprobeonasiliconsubstrate.Theoutofcontactportio$
$contactportioninred, highlightingthechangeintheresonantpeaklocationsandshapesbetweenthesetwostagesofthemeasurement.$

3. Elastic Modulus Determination of PEO

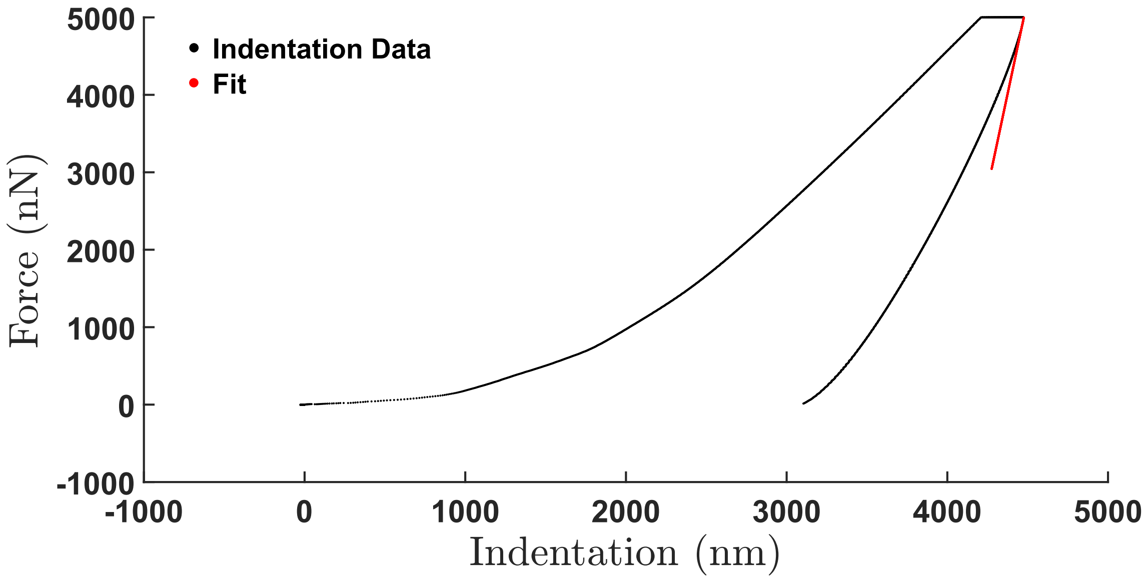

Fig. S3. anoindentation Example Elastic unloading curves for the PEO sample obtained from nanoindentation experiments with a Berkovich indenter. Slope of curve in red is the linear fit used to determine the Young's modulus of the sample.